

1.3

Are All Functions Created Equal?

Comparing Multiple Representations of Functions

LEARNING GOALS

In this lesson, you will:

- Identify equivalent forms of functions in various representations.
- Model situations using tables, graphs, and equations.
- Use functions to make predictions.
- Determine whether two forms of a function are equivalent.

KEY TERMS

- relation
- function
- function notation

Every year during the first full week in August, the residents of Twinsburg, Ohio literally see double! That's because Twinsburg hosts the annual Twin Day Festival. It is the largest gathering of twins in the world, with thousands of twins, triplets, and multiple-birth families converging on the town for a weekend of games and activities. Although twins develop their own unique personalities, they often stand out in a crowd. It might be an interesting experience for twins and non-twins alike to be in a town completely filled with groups of people who look the same. *Not* having a person who looks just like you might actually make you stand out in the crowd.

Twins only account for about 1% of the pregnancies in the world, but the number of twin births actually varies depending on where you live. For example, the rate of twin births in Massachusetts is much higher than the rate in New Mexico. The highest rates in the world are found in central Africa while the lowest rates are found in Asia.

What do you think might account for differences throughout the world in the rate of twin births? Have you ever known twins? Would you like to have a twin brother or sister?

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PROBLEM 1 You Aren't Looking Like Yourself Today

Understanding patterns not only gives insight into the world around you, it provides you with a powerful tool for predicting the future. Pictures, words, graphs, tables, and equations can describe the exact same pattern, but in different ways.

A relation is a mapping between a set of input values and a set of output values. In the problem, *The Cat's Out of the Bag*, you used a visual model, graph, table, and context to describe the relation between the number of ballot counters, and the total number of seniors that learned the result of the homecoming king election. In relations such as this one, there is only one output for each input. This type of relation is called a *function*. A **function** is a relation such that for each element of the domain there exists exactly one element in the range. **Function notation** is a way to represent functions algebraically. The function $f(x)$ is read as "f of x" and indicates that x is the input and $f(x)$ is the output.



Directions: Cut out the relations provided on the following pages. You will encounter graphs, tables, equations, and contexts. Analyze and then sort the relations into groups of equivalent representations. All relations will have at least one match.

Attach your groupings on the blank pages that follow the cut-out pages. Then provide a brief rationale for how you grouped each set of relations.

Remember that the domain is the set of all the input values and the range is the set of all the output values.

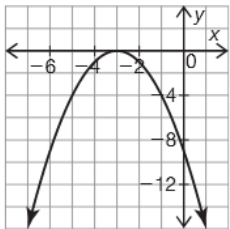
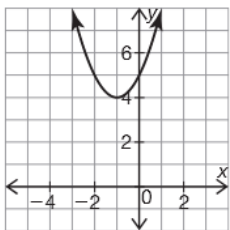


Be careful— all groupings do not necessarily have the same number of representations. Also, remember that equations can be written in different forms and still be equivalent.



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A 	B $f(x) = x^2 + 2x + 5$												
C <table border="1" data-bbox="438 817 614 1131"><thead><tr><th>x</th><th>y</th></tr></thead><tbody><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>6</td></tr><tr><td>4</td><td>8</td></tr><tr><td>5</td><td>10</td></tr></tbody></table>	x	y	1	2	2	4	3	6	4	8	5	10	D $f(x) = x^2 + 6x + 5$
x	y												
1	2												
2	4												
3	6												
4	8												
5	10												
E 	F $f(x) = -(x^2 + 6x + 9)$												

G $f(x) = 2x$	H $f(x) = (x + 5)(x + 1)$												
I $f(x) = -(x + 3)(x + 3)$	J A relation with a line of symmetry at $x = -3$, a vertex that is a maximum value, and a graph that opens down.												
K Louise heard a rumor. She tells the rumor to two people the next day. The two people that she told then tell two more people the following day, who each then go on to tell two more new people the rumor the following day. The relationship between the days that have passed and the number of new people who hear the rumor that day.	L <table border="1"><thead><tr><th>x</th><th>y</th></tr></thead><tbody><tr><td>-3</td><td>8</td></tr><tr><td>-2</td><td>5</td></tr><tr><td>-1</td><td>4</td></tr><tr><td>0</td><td>5</td></tr><tr><td>1</td><td>8</td></tr></tbody></table>	x	y	-3	8	-2	5	-1	4	0	5	1	8
x	y												
-3	8												
-2	5												
-1	4												
0	5												
1	8												

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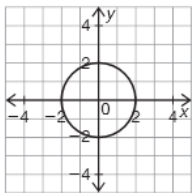
M

x	y
-4	-1
-3	0
-2	-1
-1	-4
0	-9

N

$$y = 2^x$$

O



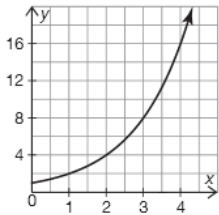
P

$$y = (x + 3)^2 - 4$$

Q

x	y
0	1
1	2
2	4
3	8
4	16

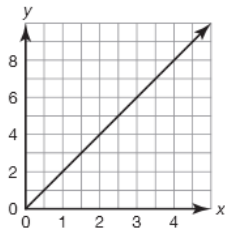
R



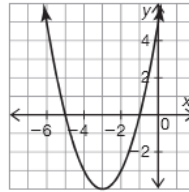
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S



T



U

Erika is worried that her secret got out. On the first day she and her best friend were the only people who knew about the secret. But now, two new people are finding out the secret every day. The relationship between the number of days that have passed and the total number of people who know about her secret.

V

$$x^2 + y^2 = 4$$

W

x	y
-5	0
-4	-3
0	5
1	12
2	21

X

x	y
-1	1.73
-1	-1.73
0	2
0	-2
1	1.73
1	-1.73

1. What strategies did you use to sort the representations into your groups?

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Did you come up with more than one way to show that different representations are equivalent?



2. How do you know which relations are functions and which are not functions? Explain your reasoning in terms of the graph, table, and equation.



3. Identify the function family associated with each grouping. How can you determine the function family from the graph, table, context, and the equation?

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PROBLEM 2 Why Are You So Square?

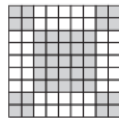


A ceramic tile company creates a new line of decorative kitchen and bathroom tiles. The company will sell larger tiles that are created from combinations of small gray and white square tiles. The designs follow the pattern shown.

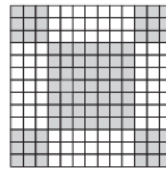
Design 1



Design 2



Design 3



1. Analyze the tile designs. Describe all of the various patterns that you notice.



2. Numerically organize the pattern.

Design Number	1	2	3	4	7	10	
Number of White Tiles, $w(n)$							
Number of Gray Tiles, $g(n)$							
Total Number of Tiles, $t(n)$							

3. What new patterns do you notice?

Don't worry about the last column for now. You will determine an expression for each type of tile later.



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4. How many total tiles are in Design 7? How many of the tiles are white? How many are gray? Explain your reasoning.



5. A hotel would like to order the largest design possible. They have enough money in their budget to order a design made up of 1700 total gray and white tiles. Which design can they afford? How many tiles in the design will be white? How many will be gray? Explain your reasoning.

6. Complete the last column of the table in Question 2 by writing an expression to describe the number of white tiles, gray tiles, and total tiles for Design n .

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7. Tonya and Alex came up with different expressions to represent the number of gray tiles in each pattern. Their expressions are shown.

**Tonya**

$$4n^2 + (2n + 1)(2n + 1)$$

Alex

$$(4n + 1)^2 - 4n(2n + 1)$$



Tonya claims that they are the same expression written different ways. Alex says, "One expression has addition and the other has subtraction. There is no way they are equivalent!"

Who is correct? Justify your reasoning using algebraic and graphical representations.



You may have noticed several patterns in this sequence. An obvious pattern is that the sum of the white tiles and gray tiles is equal to the total number of tiles. This pattern is clear when analyzing the values in the table. However, adding $w(n)$ and $g(n)$ creates a brand new function that looks very different from the function $t(n)$.

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In order to prove that the sum of the white tiles and gray tiles is equal to the total number of tiles, you must show that the expressions are equivalent.

$w(n) + g(n)$	$t(n)$
$4n(2n + 1) + (2n + 1)^2 + 4n^2$	$(4n + 1)^2$
$(8n^2 + 4n) + (4n^2 + 4n + 1) + 4n^2$	$(4n + 1)(4n + 1)$
$16n^2 + 8n + 1$	$16n^2 + 8n + 1$

8. Analyze the context, table, and expressions in this problem.

- a. Identify the function family that describes the pattern for the number of white tiles. Explain your reasoning.

- b. Identify the function family that describes the pattern for the number of gray tiles. Explain your reasoning.

The best mathematicians work in teams to prove ideas. People often point to Andrew Wiles as an interesting exception, because he worked in his attic by himself for many years to prove Fermat's Last Theorem! In truth, though, even he occasionally collaborated with a friend. Look it up sometime—it's a fascinating story!



- c. When you add the functions that represent the number of gray tiles and white tiles, does the new function belong to the same function family? Explain your reasoning.

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9. Describe the relationship between the number of white tiles and gray tiles in each design. Prove that this relationship exists.

There are many ways to prove something. Mathematical proofs consist of equations, written arguments, pictures, and flow charts. Use correct terminology to describe mathematically why you know something is true.

10. Analyze the tile patterns.
- a. Prove that the number of white tiles is always an even number.



- b. Prove that the total number of tiles is always an odd number.



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Talk the Talk

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Choose a word that makes each statement true. Explain your reasoning.

always sometimes never

1. Two functions are _____ equivalent if their algebraic representations are the same.

You can use a word more than once, or not at all. Choose wisely!



2. Two functions are _____ equivalent if they produce the same output for a specific input value.

3. Two functions are _____ equivalent if their graphical representations are the same.



Be prepared to share your solutions and methods.